Teaching Statement

Daniel Estévez

During my time as a Ph.D. student, I have taught the problem classes in the following courses in Universidad Autónoma de Madrid:

- Spring 2014: Analysis II. Degree in Physics, 1st year.
- Spring 2015: Analysis II. Degree in Physics, 1st year.
- Spring 2016: Linear Algebra. Degree in Mathematics and Computer Science, 1st year.
- Spring 2016: Differential Equations (in English). UAM-Boston Univesity international program.
- Fall 2016: Algebra I. Degree in Physics, 1st year.
- Spring 2017: Linear Algebra. Degree in Mathematics and Computer Science, 1st year.

Also, since the academic year 2012–2013 I have being organising the "Advanced problems sessions" with my thesis advisor Dmitry Yakubovich. The goal of these sessions is to teach undergraduate students from the degree in Mathematics and other similar degrees how to solve olympiad style problems.

The problem classes in Universidad Autónoma de Madrid are taught with groups of 40 to 50 students and the goal is to teach the students how to solve exercises and problems related to the syllabus. Many of the exercises are routine and others require some ingenuity by the students. The final exam and intermediate tests for the course usually consist of solving similar problems.

The Differential Equation course I taught is part of an international program between Universidad Autónoma de Madrid and Boston University. Around 40 students from the College of Engineering in Boston University come to Madrid for a semester to study several courses in Mathematics and Sciences: Differential Equations, Contemporary Physics and Electronic Circuits are some of the possible courses they can choose to study. All the courses of the international program are taught in English (other undergraduate courses in Universidad Autónoma are usually in Spanish).

The Differential Equations course is an introductory course to ordinary differential equations and it has an applied focus. In particular, there is much work done with the computer in the form of numerical simulations. Apart from teaching the students how to solve problems, one of my duties as a teacher in this course was to organise and grade the so called "Labs". These are projects that involve applying the material learnt in the course to some real life problem or situation and writing a report addressing several questions about the problem. For instance, in one of the Labs students were told to to apply the logistic model to oil production in the US and try to get some conclusions about oil shortage from this model.

The focus in these Labs was to make the students reason, as many of the times there is not a single correct answer to the problems, and several answers can be acceptable if accompanied by adequate reasoning and evidence. Another goal was to teach the students how to write reports properly.

Since my students in this course were studying engineering majors, I tried to make them aware of the issues that appear when applying mathematics to the real world. For instance, one of the Labs told students to study RLC circuits. These are simple electrical circuits formed by a resistor, an inductor and a capacitor, and their behaviour can be modelled by a damped harmonic oscillator. The book proposed some values for the components and asked students to predict the behaviour of the circuit. The problem was that the proposed value for the inductor was 1H. I told the students that an inductor of such a high value is quite bulky and expensive, and only used in specialised applications. An off-the-shelf inductor that one may find in a consumer device or an electronics lab will have an inductance on the order of uH or mH's (i.e., between 10^{-6} and 10^{-3} times smaller from what the book proposed). I think that it is important for engineering students to be aware of these aspects.

The Analysis II course that I have taught twice to Physics degree students is an introductory course to calculus in several variables. It covers from limits and continuity in $\mathbb{R}^n$ to Gauss theorem, Stokes theorem and the divergence theorem. The Linear Algebra and Algebra I courses cover essentially the same material: linear spaces and maps, determinants, dual spaces, diagonalization and Jordan normal formal.

When teaching the problem classes I have found that it is very important to try to make the students solve problems on their own at home. When doing this, they realise what things they know well and what they do not. The final exam for the course consists of solving problems of the same type. The students know and understand this, but they are usually quite busy, so many times they do not solve as many problems as they should. I think that it is good to make them engaged with problem solving in an emotional manner, usually by presenting them the problems as a personal challenge.

Another important aspect is to balance the difficulty. Most of the exercises we solve are routine, and of course I have to make sure that all the students are able to solve at least routine exercises. However, the best students can become bored and demand more difficult problems. Every time that I solve an easy problem and I see there is a variation that turns it into a more interesting problem, I announce this in class with the hopes that the best students might take solving it as a challenge. Then, they can approach me later if they run into difficulties or they manage to solve it.

I also think that it is important to relate the material that appears in class to applications in Science or Engineering. This makes students realise that what they are studying is actually meaningful. In the grand scheme of things, the courses I teach are very basic and introductory. They are only a step in the students' formation, but if looked from the correct perspective, they are the basis for everything that will come later. If an exercise in calculus is to draw the surface $x^2 + y^2 = t^2 - 1$ (a two-sheeted hyperboloid), after finishing the exercise, I will tell my Physics students that these sort of surfaces appear in Special Relativity, in the context of Lorentzian spacetime (although spacetime has 4 dimensions, but here we only have 3 to be able to draw on the blackboard). In fact, one can begin to understand relativistic effects such as time dilation from the shape of that surface. I do not try that my students learn anything about Special Relativity with this explanation, but rather that they get some interest in learning how to draw surfaces. Probably they started studying the Physics Degree to learn about relativity and such things, and not to draw surfaces, but it is good that they see this as a necessary step in their learning process. The most curious students can look up or ask me what is the relation of this hyperboloid with the Lorentz space and get a glimpse of how this works.

In Universidad Autónoma de Madrid (as in other universities in Spain), the teachers are available for office hours outside of the regular teaching hours. Students can appoint a meeting with his teacher during office hours to solve whatever individual questions or difficulties they may have with the subject. I find that these are very useful for students, particularly for students who may be very hardworking but are not so fast at grasping things the first time they see them. They may need some additional support and the office hours are perfect for that. Thus, I encourage my students to come to my office anytime they need.

During these office hours, I find that sometimes it is just necessary to repeat whatever explanation I might have told during the class, but more slowly and checking that the student understands it. Other times, a different explanation is needed. In these occasions, I try to go out of the standard way and give an explanation that, while it may not be completely correct from the mathematical point of view, can give some intuition or insight into the problem (of course I tell the students that this is not a proper mathematical explanation).

For instance, it seems that my intuitive explanation about Stoke's theorem has become quite popular among students. I tell them that if you are stirring a cup of tea, you can calculate how the liquid swirls on the surface in two different ways: you can calculate how the liquid runs along the border of the cup or you can add up all the infinitesimal swirls about each point of the surface. Stoke's theorem says that these two ways give you the same result. While this may not be mathematically correct without a proper formalisation of the setting, at least it gives some insight into how the theorem works and the physical motivation that led to the theorem originally. Without this, the theorem is just seen by students as some formal manipulation that helps them solve certain integrals. This explanation is far from being original and I probably read it somewhere, but it is a good example of the intuitive explanations I like to tell. Most of the times, I come up with my own ad-hoc explanations.

The "Advanced problems sessions" is something that I have being organising for several years now. It is completely independent of the undergraduate program and undergraduate students join the sessions voluntarily (there is no grading and students get no credit for attending the sessions). In these sessions, we pose the students problems of olympiad type, which are not straightforward to solve and usually require a lot of ingenuity. The main goal of these sessions is that interested students can work on these sort of problems, as most of the problems they solve in the courses of the undergraduate program are more simple and straightforward.

At the end of the academic year we make a test for the students and we select the best one or two students so that our Department can fund their travel to an international olympiad during the summer. Usually, the olympiad we choose is the International Mathematics Competition for University Students, which most of the years is held in Blagoevgrad, Bulgaria, and it is organised by University College London and the American University in Bulgaria.

When I was an undergraduate student, I attended these sessions for three years and I went to the international competition twice. I have been organising the sessions with my thesis advisor, Dmitry Yakubovich, since I started studying the master's degree.